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AN INVESTIGATION OF THE TRANSFER OF MONOPULSE TRACKING BETWEEN TWO COHERENT POINT SOURCES

by

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ABSTRACT

This investigation deals with the effects of phase and amplitude differences between two coherent point source radiators on the tracking behavior of a monopulse radar. The tracking function of the radar is modelled in a single plane containing two point sources located at a finite distance from the radar antenna. The complex illumination of the antenna aperture by the point sources and the resulting monopulse sum and difference antenna responses are derived, including the effects of finite range. The monopulse discriminator characteristic is formed using the antenna responses. A set of calibration curves are presented which show the transfer of the tracking point of the radar from one source to the other as the phase and amplitude differences between the sources vary. The application of this investigation is in specifying the excitation of two point sources to move the tracking point of a radar smoothly and continuously from one source to the other, in order to simulate continuous target motion using discrete positioned radiating elements.

RESUME

Cette étude traite des effets de différences de phase et d'amplitude entre deux sources ponctuelles cohérentes sur le fonctionnement de la poursuite d'un radar monopulse. Le fonctionnement de la poursuite du radar est modelé dans un seul plan contenant deux sources ponctuelles situées à une distance finie de l'antenne du radar. L'illumination complexe de l'ouverture de l'antenne par les sources ponctuelles ainsi que les réponses somme et différence de l'antenne monopulse sont dérivées, incluant les effets d'une distance finie. La charactéristique du discriminateur monopulse est formée en utilisant les réponses de l'antenne. Des courbes de calibration sont présentées montrant le transfert du point de poursuite du radar d'une source à une autre selon que les différences de phase et d'amplitude entre les deux sources varient. L'application de cette étude est dans la spécification de l'excitation des deux sources ponctuelles afin de bouger le point de poursuite d'un radar, sans à-coups et uniformément d'une source à l'autre, afin de simuler un mouvement de cible continu en utilisant des sources positionnées discrètement.

EXECUTIVE SUMMARY

There is a requirement to simulate electronic warfare engagements for the purpose of testing and validating equipment, because the cost and complexity of producing a full battle scenario is prohibitive. This problem may be approached in a number of ways, involving software, hardware, or a combination of both. A possible method of testing ECM techniques against tracking radars and related equipment is by simulating the radiated RF signals which would be encountered in a battle scenario. The essence of the simulation is to present the radar with targets, representative of the battle scenario in frequency, range, bearing and elevation, and then observe the radar's tracking behavior. To avoid interference by and with other signals which may be present in the test area, and anechoic chamber may be used to contain and absorb the RF signals radiated during the simulation.

This paper deals in a general way with the problem of generating target motion as seen by a radar in an anechoic chamber, during such a simulation. Specifically, the radar is positioned at one end of the chamber, while the opposite end of the chamber holds an array of radiating elements. A radiating target may be given the illusion of motion, as seen by the radar, by sequentially switching on and off adjacent elements. The effect is similar to the travelling lights of a movie marquee sign. In the simplest case, as one element is switched off, the adjacent element is immediately switched on. Repeating this procedure (in the same direction) gives the illusion of a travelling target. The motion is not continuous using this simple approach - the element appears to jump instantaneously from one element position to the adjacent one. This may be undesirable under certain circumstances, and is certainly an approximation to a real scenario, where targets move continuously in angle.

It is theoretically possible to approximate continuous target motion without abandoning the concept of an array of elements. By carefully controlling the amplitude and phase between two adjacent elements, it is possible to smoothly and continuously move the apparent target position between two element positions. This paper investigates the effects of different phase and amplitude relationships between two target elements, on the tracking point of a monopulse radar.

In the body of the analysis, simple mathematical models are developed representing the radar and the point source targets, in the azimuth plane. These models are used to produce a series of azimuth plane calibration curves for determining phase and ampliutde settings of the two elements, when manipulating the tracking point of the radar between them. The issues of realizing a hardware system capable of using such calibration curves are beyond the scope of this paper.

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1.0 INTRODUCTION

This investigation deals with the effects of a pair of coherent, different amplitude point sources on the tracking of a monopulse radar. The point sources represent radiating dipole elements in a one dimensional or two dimensional array mounted in an anechoic chamber. In the context of this investigation, these radiating elements are used to represent moving targets to a radar at the opposite end of the anechoic chamber. The illusion of a moving target as seen by the radar may be simulated in a number of ways using these radiating elements. Perhaps the simplest way is by sequentially activating adjacent radiating elements (such as dipoles) along the desired trajectory in the array, with a single radiating element representing a single target to the radar.

In this simple case, the quality of this array approximation of continuous target motion depends on, among other things, the angular density of the array as seen by the radar. With only a single dipole activated (per target) at a time, the target may only take on discrete angular values corresponding to the positions of the elements in the array. For this controlling scheme, the denser the array the better the approximation will be to continuous target motion.

It is possible to increase the effective density of the array by activating a number of radiating elements simultaneously to represent a single target to the radar. This is appealing because although the control of the active elements is more complicated, it does not involve the increased hardware cost and complexity of physically adding more elements. The simplest case of single target synthesis using multiple radiators is the use of two dipoles to form a single target, thus synthesizing a single virtual element whose position may be controlled either continuously or discretely by adjusting the signals radiated by the pair of elements. Manipulation of the position of the virtual element to discrete positions between the two radiating elements can increase the effective array density by the number of these discrete positions. Continuous control of the virtual element position is the limiting case of discrete control and is, in some sense, the ideal of synthesizing the geometry of target motion. Broadly speaking, there are two approaches to achieve this - one involves radiating coherent signals from the two elements, the other involves radiating noncoherent signals.

One approach synthesizing a virtual element by radiating noncoherent signals involves a deliberate frequency difference between the two radiating elements. The frequency of one element may be offset from the other by some small amount such that both elements are in the pass band of the radar, and the beat frequency between them is not detectable by the signal processing (eg. AGC circuits) of the radar. Equal amplitude signals from the two elements may place a virtual element half way between the two elements (ie. at the power centroid between the elements, since no coherent radiation pattern is produced by the elements). Adjusting the relative amplitude of the elements can bias the position of the virtual element toward one element or the other. A detailed analysis of the way the radar discriminator is formed is necessary to predict exactly how the tracking point is transferred from one element to the other. There is some evidence to suggest that this technique may not work for monopulse radars and steady amplitude noncoherent sources because of the normalization used in forming the discriminator [1].

In the coherent case, signals from the two elements are at exactly the same frequency, and so the pair form a coherent radiation pattern. The *radiation pattern of a pair of point source radiators is known as an* interferometer pattern. The form of the radiation pattern depends on the relative amplitude weighting of the elements, while phase adjustments have the effect of steering the pattern within the envelope of the element factor (radiation pattern of a single element). The tracking point of the radar, ie. the position of the virtual element, is therefore a function of the relative phase between the elements as well as the relative amplitude. Additionally, it is important to consider possible effects of the finite separation between the radar antenna and the two radiating elements, since Fresnel region effects on radar tracking performance may be significant. The joint Fresnel region of the radar antenna and a pair of point sources may be taken as $(D+d)^2/\lambda$, where D is the dimension of the radar antenna and d is the separation of the two radiating elements. If d is small, the Fresnel region may approximated as D^2/λ (see Appendix I).

Adjusting the relative phase between the radiating elements translates or steers the interferometer pattern in angle, which generally changes the electric field illumination intercepted by the radar antenna aperture. The larger the separation of the two elements as seen by the radar, the more sensitive will be the aperture illumination to phase changes between the elements. This effect is important because the position of the virtual element as seen by the radar is a function of the aperture illumination. In an extreme case, for example, if the two elements are 180° out of phase as seen by the radar, a crosseye effect appears in the tracking behavior, establishing a pair of virtual elements (rather than a single virtual element), one to either side of boresight, located well to the left and right of the pair of radiating elements (assuming an azimuth separation of the two radiating elements). For the equal amplitude case, a 0° phase difference between the elements as seen by the radar produces a virtual element half way between the two radiating elements (though other combinations of amplitude and phase can produce a virtual element in the same position). In the general case of unequal amplitude and non-zero phase difference, the position of the virtual element no longer corresponds to the power centroid of the two radiating elements.

If the radar beamwidth is narrow relative to the angular separation of the two elements, then the position of the virtual element can become a sensitive function of the relative phase between the elements. Even with perfect phase control between the elements, short range can affect the tracking performance of the radar in the sidelobe regions (see Appendix II). This is because the electric field variation over the antenna aperture may not be uniform in amplitude and phase. If the angular separation of the radiating elements is large as seen by the radar, the amplitude of the aperture illumination may be non-uniform due to the angular variation of electric field strength of the two elements. This effect may be aggravated if the radiation pattern of the two elements is improperly steered, due to phase errors between the elements. Additionally, non-uniform phase distribution may arise across the aperture if the range between the radar and the elements is not large enough (ie. if the radiating elements are within the Fresnel region of the radar antenna). Uniform aperture illumination in both phase and amplitude is a fundamental assumption for proper radar tracking of a target in the far field of the radar, and with the radar in the far field of the target.

The principal advantage of the noncoherent approach over the coherent approach described above is that only the relative amplitude of the two radiating elements needs to be controlled in establishing or shifting the position of a virtual element. However, this approach cannot be used to represent targets to a coherent radar, and a careful study must be made of the signal processing of the noncoherent radar to determine the acceptable beat frequency between the elements. A careful study must also be made in the noncoherent case of the way the discriminator is formed in the radar, since this may reveal peculiarities in the manner that the radar treats two unresolved targets. For instance, monopulse radars in particular may not track the power centroid of two steady amplitude unresolved noncoherent sources. The coherent approach may be used with both coherent and noncoherent radars, though both the phase and amplitude of the radiating elements must be controlled.

This investigation considers the coherent approach, using pairs of elements in an array to simulate continuous target motion. The resulting radar tracking behavior is of interest in determining the shortcomings of this approach. In order to study the combined effects of finite range and amplitude and phase variation between the two elements, the range between the radar and the point sources (the length of the anechoic chamber) selected in this investigation is 13.7 m, and is slightly larger than the Fresnel region of the radar antenna, as defined in Appendix I.

The investigation involves determining the effective monopulse sum and difference antenna responses which result from illumination of the antenna aperture by two point sources at the given range. The discriminator characteristic describes the angular location of the principal tracking point of the radar when illuminated in this fashion. In order to form the effective antenna sum and difference responses from which the discriminator may be derived, the illumination of the antenna by the point sources must be modelled. A Fourier Transform relationship is used to approximate the resulting antenna responses. With this model of the discriminator characteristic, the transfer of the tracking point of the radar from one radiating element to the other may be determined, as a function of the relative phase and amplitude of the two elements.

2.0 RADAR TRACKING RESPONSE TO TWO POINT SOURCE RADIATORS

In this section the monopulse radar's tracking response to illumination by two coherent point source radiators is derived.

2.1 Derivation of the Radar Antenna Responses

The analysis is carried out using a one-dimensional (line) aperture model of the monopulse radar antenna. The radar sum and difference aperture weightings (E-field illumination of the antenna by the feed) are shown in fig. 1a, where y is the aperture domain variable. The sum aperture weighting is a uniform weighting, while the difference pattern is formed from a linear variation in amplitude, in quadrature with the sum aperture weighting. From Fourier Transform theory, this implies that the difference pattern is the

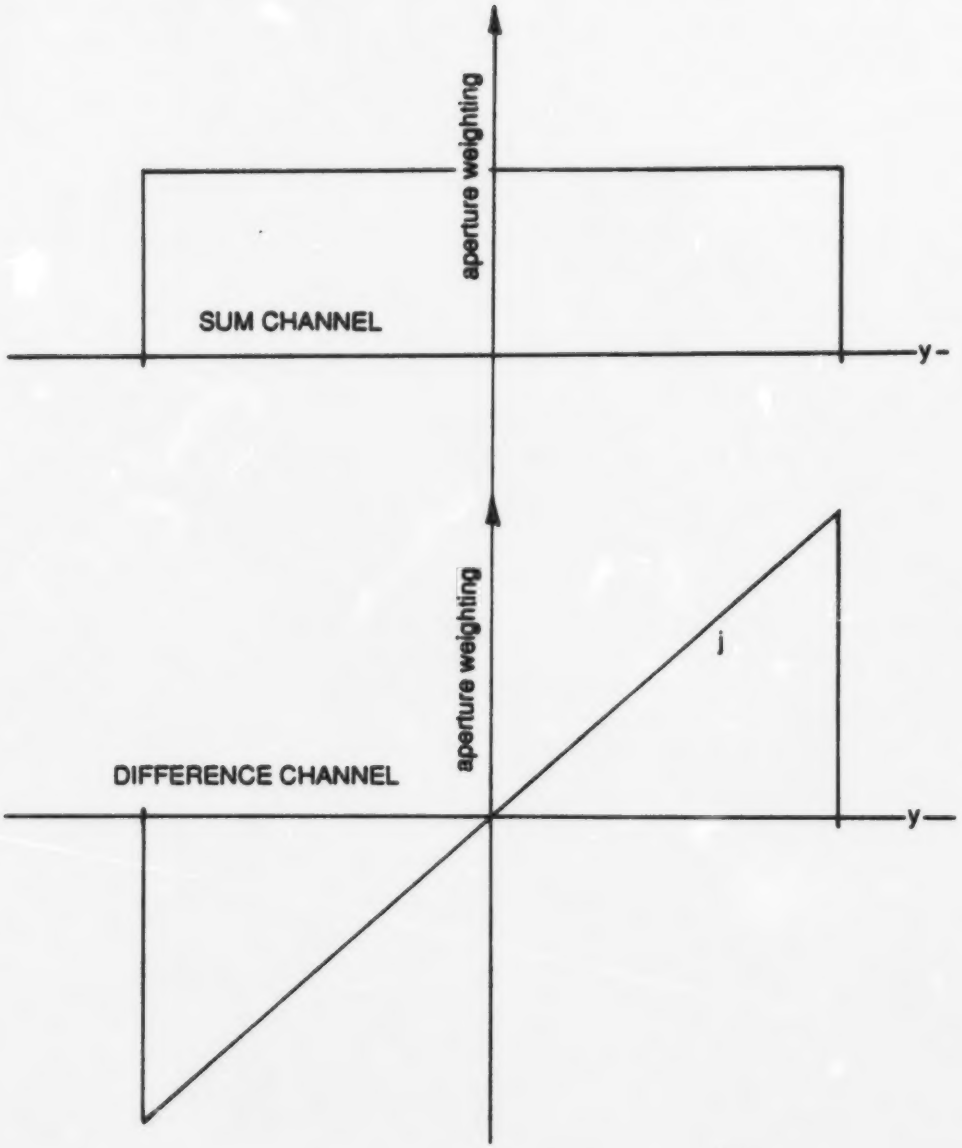

FIG. 1a: MONOPULSE SUM AND DIFFERENCE ANTENNA APERTURE WEIGHTINGS

derivative (in the Fourier Transform variable) of the sum pattern. The corresponding far field sum and difference patterns are shown in fig. 1b. The sum and difference weightings are in quadrature because the radar antenna is illuminated by a monopulse feed consisting of two radiating elements, with cophase excitation corresponding to the sum weighting and anti-phase excitation corresponding to the difference weighting.

When the radar is illuminated by a point source at the end of the chamber, the far field antenna responses are an approximation of the actual finite range antenna responses, because Fresnel region effects are ignored in the far field assumptions. In the general case, the validity of approximating the actual antenna sum and difference responses by the far field form depends on the antenna aperture size and the range to the point source illuminator. In this investigation, the antenna aperture is not actually being illuminated by a point source radiator, but by a pair of point source radiators, in the form of a pair of dipoles at one end of an anechoic chamber. However, because the length of the chamber is 13.7m and the separation of the dipoles in this investigation is small (20cm) relative to this, the pair of dipoles is considered to be a point source radiator. From Appendix I, the Fresnel region of the pair of dipoles extends to 1.2m at 9 GHz, which is less than 1/10th the length of the chamber. The diameter of the radar antenna is 63.5cm, which gives a far field region of 12.1 m.

The effective sum and difference antenna patterns in the anechoic chamber may be derived by considering the illumination of the radar antenna aperture. Fig. 2a shows the geometry for deriving the effective antenna responses.

The relative phase of the illumination over the antenna aperture is given by:

$$g(y) = \frac{2\pi}{\lambda} \left(R^2 + y^2 \right)^{\frac{1}{2}} \qquad (1)$$

where R is the range between the radar and the point source, g(y) is the phase profile over the aperture, and y is the aperture domain variable. The antenna response near boresight may be approximated by the Fourier Transform of the product of the aperture weighting and the aperture illumination, as shown below:

$$F(ky) = \int_{-a/2}^{+a/2} D(y) f(y) e^{jg(y)} e^{-jk_y y} dy \qquad (2)$$

where

a = width of the radar antenna aperture
D(y) = radar antenna aperture weighting
f(y) = (complex) radar antenna aperture illumination

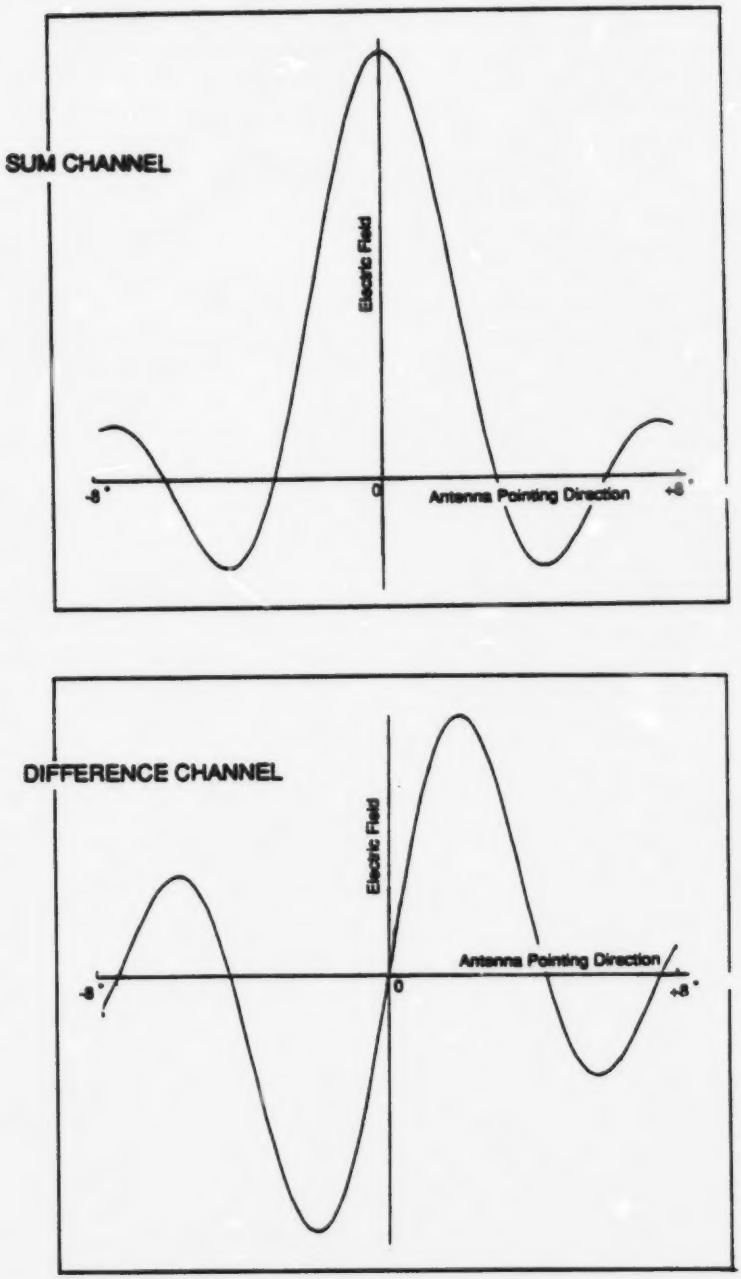

FIG 1b: MONOPULSE SUM AND DIFFERENCE ANTENNA RESPONSES

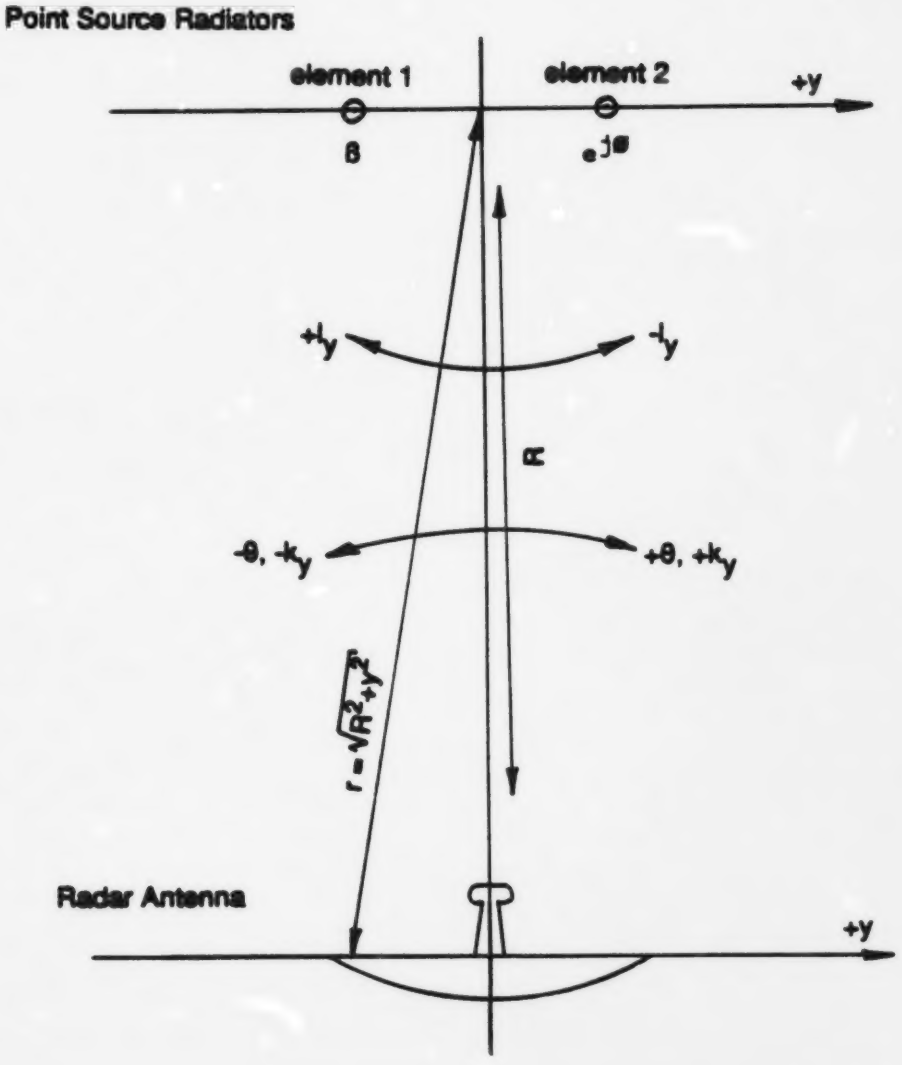

FIG. 2a: GEOMETRY OF RADAR ANTENNA AND POINT SOURCES

$e^{jg(y)}$ = phase function over the victim aperture associated with illumination by a point source radiator, and with the finite length of the anechoic chamber (see eqn. (1)).

k_y = Fourier Transform variable, $2\pi\sin\theta/\lambda$, where θ is angle measured around the radar antenna aperture

The aperture illumination and phase profile functions may be written as the sum of real and imaginary parts:

$$f(y) = f_r(y) + jf_i(y) \tag{3}$$

$$e^{jg(y)} = g_r(y) + jg_i(y) = \cos(g(y)) + j\sin(g(y)) \tag{4}$$

Using eqns. (3) and (4), the integrand of eqn. (2) can be written as the sum of real and imaginary parts:

$$F(ky) = \int_{-a/2}^{+a/2} D(y)\left[[f_r g_r - f_i g_i]\cos(k_y y) + [f_r g_i + f_i g_r]\sin(k_y y)\right] + jD(y)\left[[f_r g_i + f_i g_r]\cos(k_y y) - [f_r g_r - f_i g_i]\sin(k_y y)\right] dy \tag{5}$$

The sum channel response of the radar is given in this model by setting $D(y) = 1$, while the difference channel response is obtained by setting $D(y) = jy$.

2.2 Derivation of Radar Aperture Illumination

In order to calculate the sum and difference channel responses of the radar, an expression is required for the complex illumination $f(y)$ of the radar aperture by the two point sources at the end of the anechoic chamber. The geometry for calculating the radiation pattern of the two point sources is shown in fig. 2a. The radiation pattern of two point sources of equal amplitude is a completely real (or completely imaginary) interferometer radiation pattern, and is shown in fig. 2b for an arbitrary separation of the two sources, and ignoring the envelope of the radiation pattern of a single element. Note that equal phase excitation of the elements produces a relative maxima of electric field strength along boresight. Increasing the separation of the sources compresses the interferometer pattern in angle and increases the number of lobes in the visible region of the antenna pattern. An amplitude difference between the sources produces a complex radiation pattern, while varying the relative phase between the point sources translates the radiation pattern in angle. In either case, the far field radiation pattern of the two point sources may be expressed as a Fourier Transform:

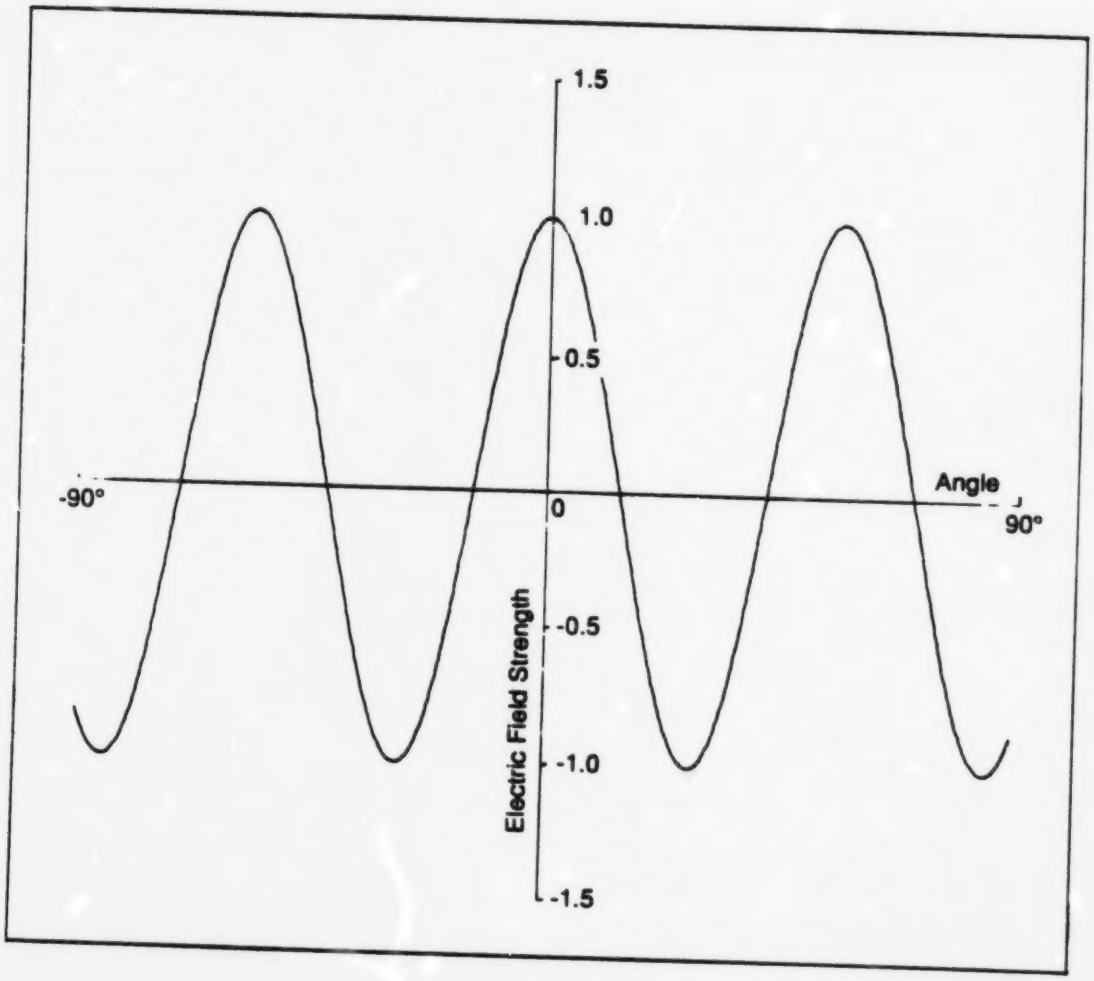

FIG. 2b: RADIATION PATTERN OF TWO COHERENT POINT SOURCES

$$I(l_y) = \int_{-\infty}^{+\infty} [\beta\sigma(y + \frac{b}{2}) + e^{j\phi}\sigma(y - \frac{b}{2})]e^{jl_y y}dy \qquad (6)$$

where

b = separation of the two point source radiators
$\sigma(y)$ = the Dirac delta function representing a point source radiator
ϕ = relative phase between the two point sources
l_y = Fourier Transform variable, $2\pi\sin\theta/\lambda$, where θ is angle measured relative to the normal of the line joining the two point sources
β = ratio of the amplitudes of the two sources, and is defined as:

$$\beta = \frac{\text{amplitude of element 1}}{\text{amplitude of element 2}} \qquad (7)$$

Eqn. (6) may be evaluated as:

$$I(l_y) = \beta\, e^{jl_y\frac{b}{2}} + e^{j(\phi - l_y\frac{b}{2})} \qquad (8)$$

Eqn. (8) can be separated into real and imaginary parts:

$$I(l_y) = \left[\beta\cos(l_y\tfrac{b}{2}) + \cos\phi\cos(l_y\tfrac{b}{2}) + \sin\phi\sin(l_y\tfrac{b}{2})\right] + j\left[\beta\sin(l_y\tfrac{b}{2}) + \sin\phi\cos(l_y\tfrac{b}{2}) - \cos\phi\sin(l_y\tfrac{b}{2})\right] \qquad (9)$$

The radiation pattern of eqn. (9) provides the illumination function $f(y)$ over the victim aperture required in eqn. (2). The transformation from the Fourier Transform variable l_y to the radar aperture domain variable y involves substituting $l_y = 2\pi\sin\theta/\lambda$ and $\sin\theta = y/R$ into eqn. (9), where R is the range between the radar antenna aperture and the point source of the illumination. The resulting expression of the illumination of the radar antenna aperture is only valid for small values of y relative to the range R,

or more specifically if the relation $1/(y^2 + R^2) \approx 1/R^2$ holds. This approximation is valid for the aperture width of 63.5 cm and the range R = 13.7 m, and is necessary because the 1/r variation of electric field strength with range from the point sources has been suppressed in eqn. (9). The approximation is linked to the relative amplitude of the fields illuminating the antenna aperture, and not to the relative phase across the antenna aperture. The phase profile across the aperture, resulting from path differences between the point source and points on the receiving aperture, is accounted for by eqn. (1).

Using the above transformation, the radar antenna aperture illumination f(y) may be expressed as:

$$f(y) = \left[\beta\cos\left(\frac{b\pi y}{\lambda R}\right) + \cos\phi\cos\left(\frac{b\pi y}{\lambda R}\right) + \sin\phi\sin\left(\frac{b\pi y}{\lambda R}\right) \right]$$

$$+ j \left[\beta\sin\left(\frac{b\pi y}{\lambda R}\right) + \sin\phi\cos\left(\frac{b\pi y}{\lambda R}\right) - \cos\phi\sin\left(\frac{b\pi y}{\lambda R}\right) \right] \qquad (10)$$

Writing the real and imaginary parts separately, the components of the aperture illumination are given as:

$$f_r(y) = \left[\beta\cos\left(\frac{b\pi y}{\lambda R}\right) + \cos\phi\cos\left(\frac{b\pi y}{\lambda R}\right) + \sin\phi\sin\left(\frac{b\pi y}{\lambda R}\right) \right] \qquad (11)$$

$$f_i(y) = \left[\beta\sin\left(\frac{b\pi y}{\lambda R}\right) + \sin\phi\cos\left(\frac{b\pi y}{\lambda R}\right) - \cos\phi\sin\left(\frac{b\pi y}{\lambda R}\right) \right] \qquad (12)$$

The radar sum and difference antenna responses due to illumination by the two point sources are given by substituting eqns. (11) and (12) into eqn. (5), with the sum response given by D(y) = 1 and the difference response given by D(y) = jy.

The monopulse discriminator characteristic is commonly defined as:

$$E(ky) = \mathrm{Re}\left[\frac{\Delta(k^y)}{\Sigma(k_y)} \right] \qquad (13)$$

where $\Delta(k_y)$ is the difference channel response and $\Sigma(k_y)$ is the sum channel response.

The monopulse discriminator may be written in terms of the real and imaginary parts of the sum and difference responses using the definitions:

$$\Delta(ky) = \Delta_r(ky) + j\,\Delta_i(ky) \tag{14}$$

$$\Sigma(ky) = \Sigma_r(ky) + j\,\Sigma_i(ky) \tag{15}$$

where eqn. (5) defines $\Sigma(k_y)$ with $D(y) = 1$ and $\Delta(k_y)$ with $D(y) = jy$. The discriminator characteristic may thus be expressed as:

$$\Sigma(ky) = \frac{\Delta_r\Sigma_r + \Delta_i\Sigma_i}{\Sigma_r^{\,2} + \Sigma_i^{\,2}} \tag{16}$$

The monopulse discriminator may be evaluated by using eqns. (5), (11), (12), (14), (15) and (16), and determines the angle of the principal tracking point of the radar, as the relative amplitude and phase between the two point sources are varied.

3.0 DISCUSSION

The location of the principal tracking point of the radar varies as the relative phase and amplitude of the two point sources change. The locus of the principal tracking point is shown in fig. 3, as the phase between the two point sources is varied from 0° to +100°, and the relative amplitude ratio varies from -30 dB to +30 dB. Curves for negative phase differences may be generated by reflecting the curves of positive phase differences about both coordinate axes.

For 0° phase difference between the two point sources, there is a symmetrical, smooth transfer of the principal tracking point of the radar from one radiating source to the other as the relative amplitude ratio varies. By referring to eqn. (7), a relative amplitude ratio less than 1 corresponds to element 2 being stronger than element 1 (see fig. 2a), whereas the opposite is true for element ratios larger than 1. Also, referring to fig. 3, extremes of relative amplitude ratio are required to move the tracking point of the radar to a good approximation of tracking either one element or the other. For 0° phase difference, a 25 dB amplitude ratio is required to bring the tracking point of the radar to within 5% of the true single source position (referenced to the distance between the two point sources). The difference between the tracking point of the radar and the actual source position at maximum or minimum amplitude ratio represents the "dead zone" over which continuous target motion may not be synthesized using this scheme. For this example, a dynamic range of 30 dB in amplitude ratio between the two point sources produces a dead zone over approximately 5% of the separation of the two

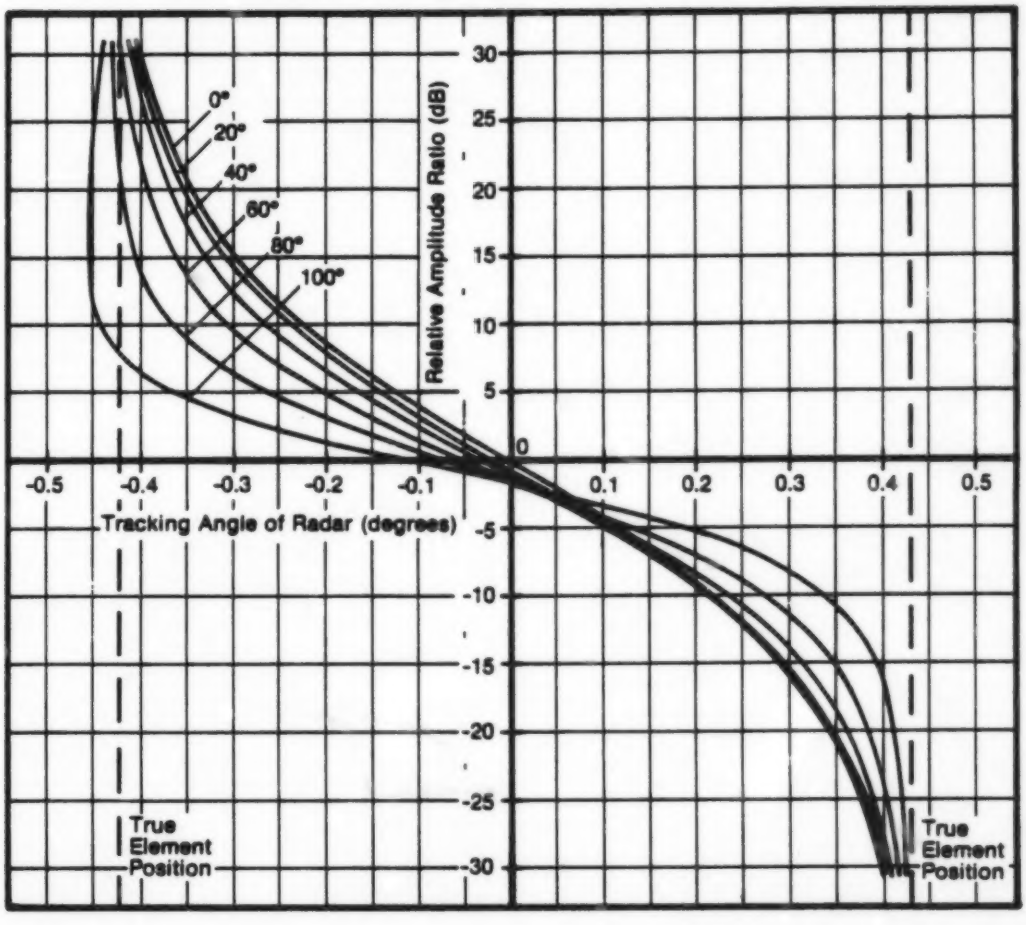

FIG. 3: LOCUS OF RADAR PRINCIPAL TRACKING POINT AS PHASE AND AMPLITUDE OF THE POINT SOURCES ARE VARIED

sources (see fig. 3). The larger the amplitude ratio variation (dynamic range), the wider the angular zone over which continuous target motion can be synthesized (ie. the smaller the dead zone).

A relative phase difference between the two point sources causes a bias in the tracking point of the radar, relative to the 0° phase difference case. For the conventions chosen in this investigation (see figs. 2 and 3, eqn. (8)) and roughly speaking, an amplitude ratio less than 0 dB biases the tracking toward element 2, with amplitude ratios greater than 0 dB producing a bias toward element 1. This generalization is not valid for amplitude ratios near 0 dB, since the sign of the phase error determines the direction of the bias in this region (see fig. 3).

This bias may be understood in terms of an asymmetry in the amplitude of the illumination f(y) of the radar antenna aperture (see eqn. (2)), since changing the relative phase between the sources produces an angular translation of the radiation pattern of the two point sources. Fig. 4 shows the real part of the illumination of the radar antenna aperture for the case of equal amplitude excitation of the two point sources for several values of phase difference. Because of the relatively short range between the two point sources and the radar antenna aperture, the illumination of the aperture is not uniform even for the 0° phase case (though it is symmetric)- there is an appreciable variation in the electric field strength across the aperture. This is the variation of electric field strength near the peak of a single lobe of the radiation pattern of the two point sources (see fig. 2b), over the angular region intercepted by the antenna aperture. At larger ranges the antenna aperture subtends a smaller angle from the point sources, and so intercepts a smaller arc of the illumination - this leads to a more uniform illumination of the antenna aperture. The principal effect of this nonuniform illumination at close range is to disturb the radar's tracking characteristics in the sidelobe regions (see Appendix II).

The asymmetry of the aperture illumination, due to either phase or amplitude differences between the two elements, produces an asymmetry in the effective antenna sum and difference patterns, accounting for the biasing of the principal tracking point of the radar. It is important to note that if the phase difference between the two sources becomes very large, gross perturbations of the antenna responses are possible, especially when the elements are nearly equal in amplitude. Fig. 5 shows the locus of the tracking point of the radar for phase differences from 0° to 180°, for the example considered in this investigation. In the extreme case, a phase difference near 180° produces a crosseye effect in the radar, moving the tracking point well outside the angular positions of the two sources. The phase difference at which the deviation of the tracking point locus from the ideal (0° case) becomes significant is not treated in this investigation, and in any case depends somewhat on the desired performance of the radar and the simulation.

Although fig. 3 is valid only for the specified source separation, radar antenna aperture width, frequency, and range between the radar and the sources, it can be used as a calibration graph in this context. Using fig. 3,

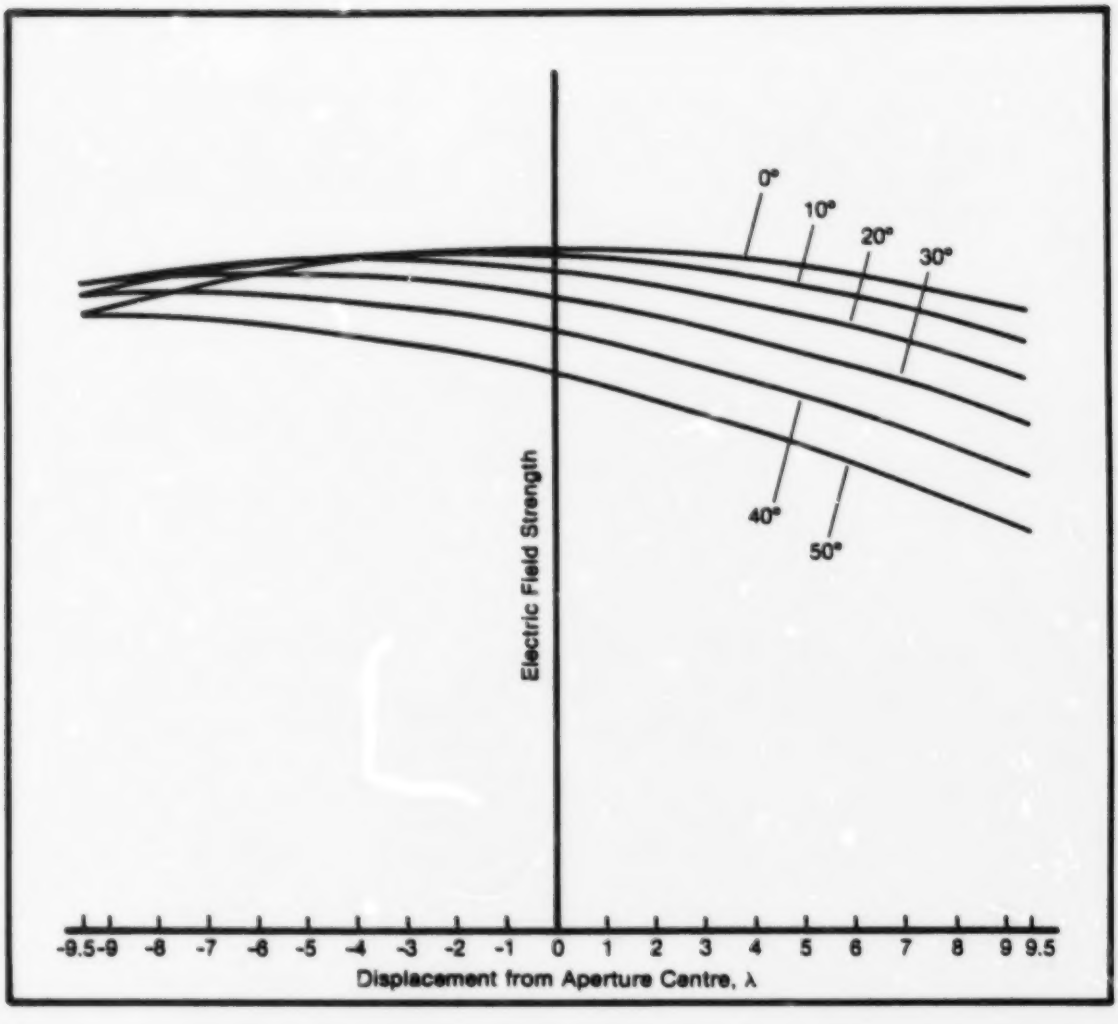

FIG. 4: RADAR ANTENNA APERTURE ILLUMINATION BY EQUAL AMPLITUDE POINT SOURCES, FOR SEVERAL VALUES OF PHASE DIFFERENCE

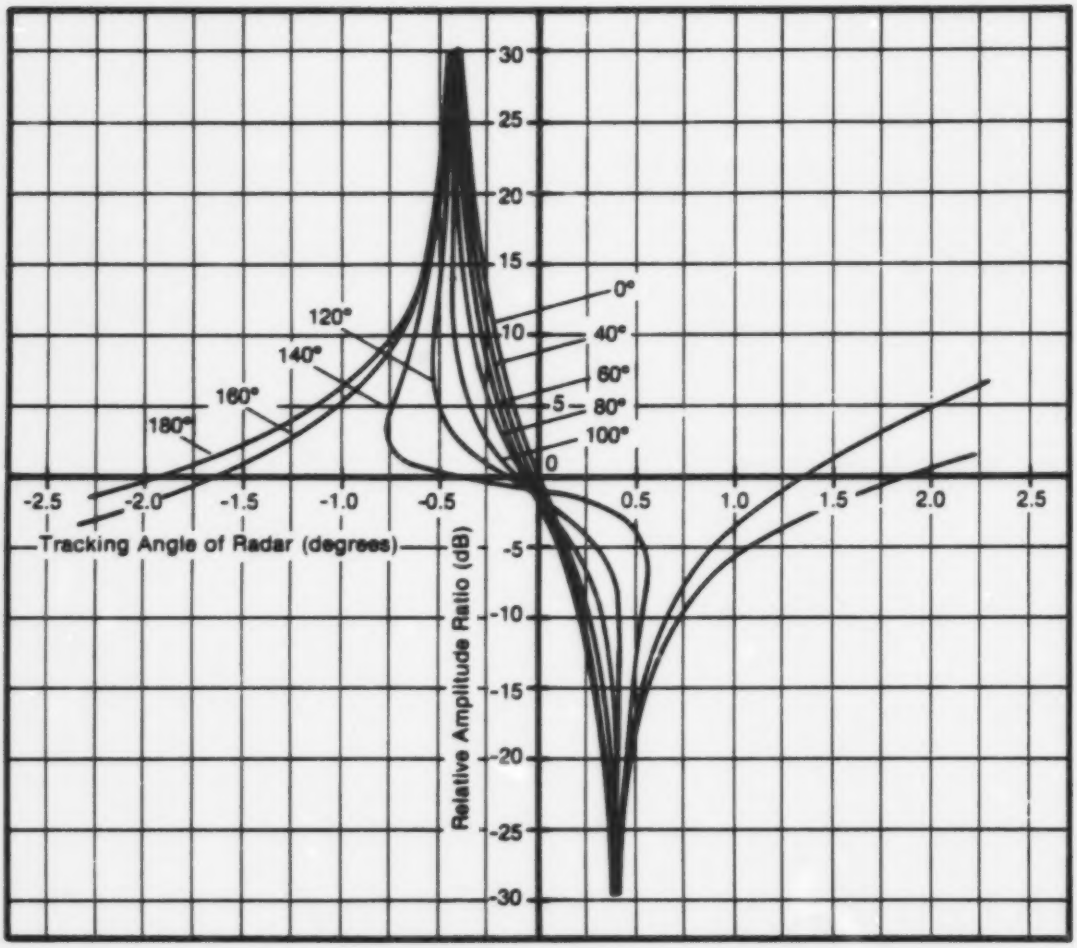

FIG. 5: LOCUS OF RADAR PRINCIPAL TRACKING POINT AS PHASE AND AMPLITUDE OF THE POINT SOURCES ARE VARIED, EXTENDED PHASE VALUES

the required relative amplitude ratio between the two point sources may be specified in order to place the tracking point of the radar at a desired point between the two point sources. The curves for phase differences between the sources indicate the expected variation of the tracking point from the 0° case, for a given phase accuracy between the two point sources.

4.0 CONCLUSION

This investigation has treated the effect of two point source radiators on the single-plane tracking of a modelled monopulse radar. Variation of the relative amplitude and phase of the two sources can be used to transfer the principal tracking point of the radar smoothly and continuously between the two sources. Equations were derived to model the radar's sum and difference channel responses to illumination by two arbitrarily excited point sources. These antenna responses were used to form the monopulse discriminator. A set of calibration curves were produced which show the locus of the principal tracking point of the radar as the relative amplitude ratio of the two sources is varied, for several values of phase between the sources. A phase difference between the two sources biases the tracking of the radar toward one source or the other, whith the direction of the bias depending on which element leads in phase.

5.0 REFERENCES

1. Constantino Melino, "Average Monopulse Angle Tracking Response to Two Unresolved Sources", IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-23, No. 5, September, 1987

APPENDIX I

DEFINITION OF THE FRESNEL REGION OF AN APERTURE ANTENNA

Figure A-1 shows a point source, located on the radar's boresight, surrounded by concentric spheres separated by $\lambda/2$ so that successive shells are 180° out of phase with each other. The intersection of these shells with the radar antenna aperture forms rings on the aperture plane, known as Fresnel zones. As the range between the aperture and the point source becomes larger, the curvature of the shells intersecting the aperture plane becomes shallower, and in the limit the aperture has only a single Fresnel zone impressed on it. This occurs at a range of $D^2/4\lambda$ from the point source, where D is the largest aperture dimension (the diameter for a circular aperture) measured in wavelengths. However, this distance cannot be taken as the far field because the edges of the aperture are 180° out of phase with the centre of the aperture. An acceptable pproximation to the far field may generally be taken as D^2/λ, a range at which the aperture edges are $\pi/4$ out of phase with the centre. D^2/λ may be taken as the range at which the point source illumination of the radar is an acceptable approximation of a plane wave.

A $\pi/8$ phase variation at the aperture edge is often used in specifying the extent of the Fresnel region, yielding a criterion of $2D^2/\lambda$. However, because the range of interest in this investigation is of the order of 12 m, the D^2/λ criterion was taken, as a matter of convenience.

For a radar aperture of 63.5 cm or 19λ at 9 GHz, the D^2/λ rule implies a Fresnel region of approximately 12.1 m.

Reference: Silver, Samuel; Microwave Antenna Theory and Design, McGraw-Hill Book Co. Inc., New York, 1949, pp 196 ff

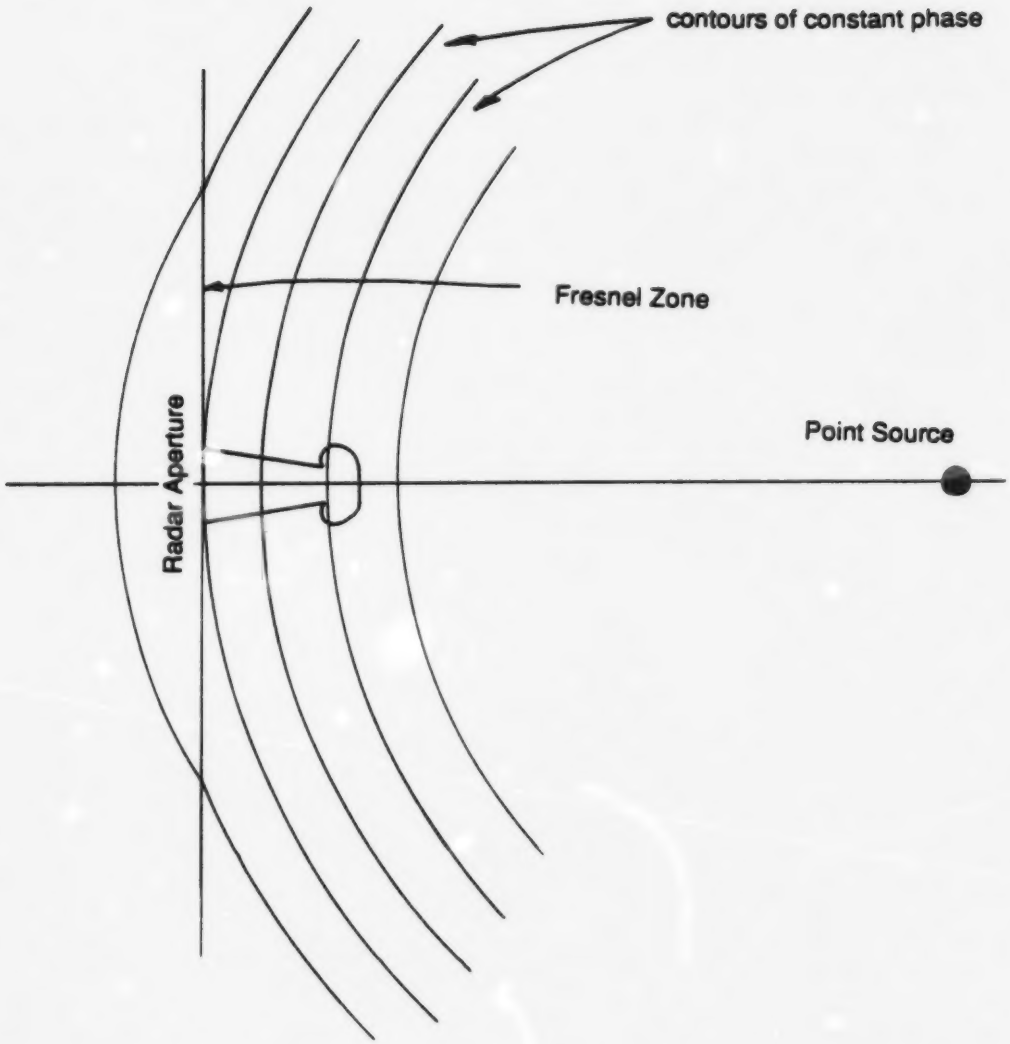

FIG. A-1: DEFINITION OF FRESNEL ZONES ON AN OPERTURE ANTENNA

APPENDIX B

EFFECT OF FINITE RANGE ON RADAR DISCRIMINATOR CHARACTERISTIC

Eqns. (4), (5), (11), and (12) may be used to determine Fresnel region effects on the sum and difference antenna responses of a monopulse radar, modeled in a single plane. The resulting effects on the monopulse discriminator characteristic may be calculated using additionally eqns. (14), (15), and (16). Although these equations are generalized to the case of two coherent point sources illuminating the radar antenna aperture, they can be used in the single point source case by setting the relative amplitude of one of the sources to zero, ie. set $\beta = 0$ (where β is defined by eqn. (7)). Programs to calculate the sum and difference responses and the discriminator characteristic using these equations are included in this appendix.

The antenna responses and discriminator characteristic were calculated for the case of an aperture width of 19λ and ranges of 10,000 m (far field), 13.7 m, and 9.1 m measured at 9 GHz. The sum aperture weighting is uniform in phase and amplitude, whereas the difference aperture weighting is a linear variation in amplitude across the aperture, zero valued at the centre of the aperture and in quadrature with the sum aperture weighting. No attention has been given to the relative scaling of the sum and difference aperture weightings, as the form of the antenna responses and discriminator characteristic is of interest rather than the actual values.

The squared magnitude of the sum channel response to illumination by a single point source at the various ranges is shown in fig. B-1. The principal effects of finite range on the antenna response is to lower the gain of the response and raise the level of the first sidelobe. Also, the first nulls (to either side of boresight) disappear. These sidelobe effects appear to be the most significant feature of the distortion of the antenna response by finite range between the aperture and the single point source.

The squared magnitude of the difference channel response to illumination by a single point source at the various ranges is shown in fig. B-2. The principal effects of finite range on the antenna response is to lower the gain of the lobes of the response closest to boresight, and raise the level of the next sidelobe to either side of boresight. Also, the first nulls (to either side of boresight) disappear. As with the sum channel response, these sidelobe effects appear to be the most significant feature of the distortion of the antenna response by finite range between the aperture and the single point source.

The discriminator characteristic of the monopulse radar resulting from illumination by a single point source at the various ranges is shown in fig. B-3. The region near boresight is relatively unaffected by the range between the antenna and the point source illuminator. However, as was observed for the sum and difference responses, the structure near the first sidelobe region is disturbed. The first spurious tracking point corresponding to this sidelobe (on either side of boresight) is translated closer to boresight as the range is decreased. A similar effect is observed regarding the next sidelobe, though to a lesser extent. Also, the slope of the discriminator characteristic is reduced in the first sidelobe region as the range is decreased.

At the ranges considered here, finite range effects do not appear to seriously affect the radar discriminator characteristic, with the possible exception of the first order sidelobe regions. If the tracking behavior of the radar in these regions is of interest, this analysis indicates that it may be appropriate to quantify the aberration of the discriminator characteristic from the far field form.

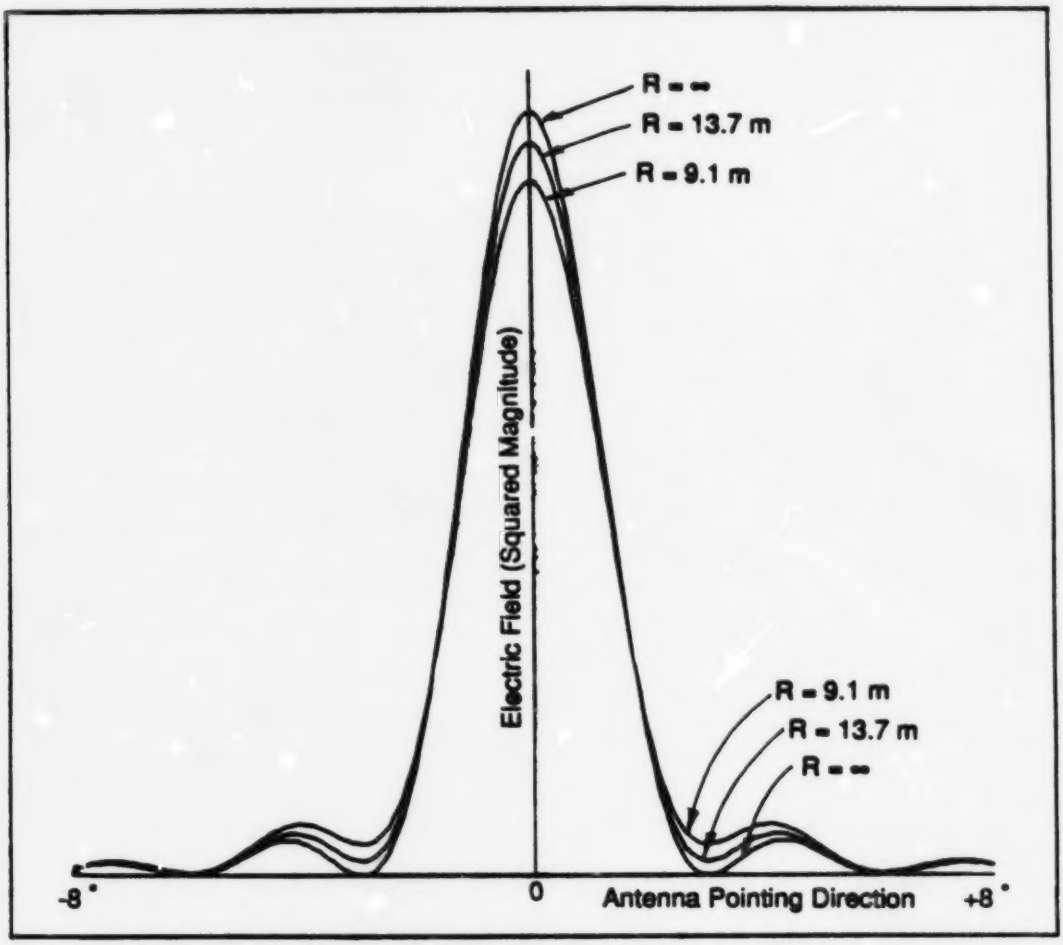

FIG B-1: SQUARED MAGNITUDE OF THE SUM CHANNEL RESPONSE AT VARIOUS RANGES BETWEEN RADAR AND A SINGLE POINT SOURCE

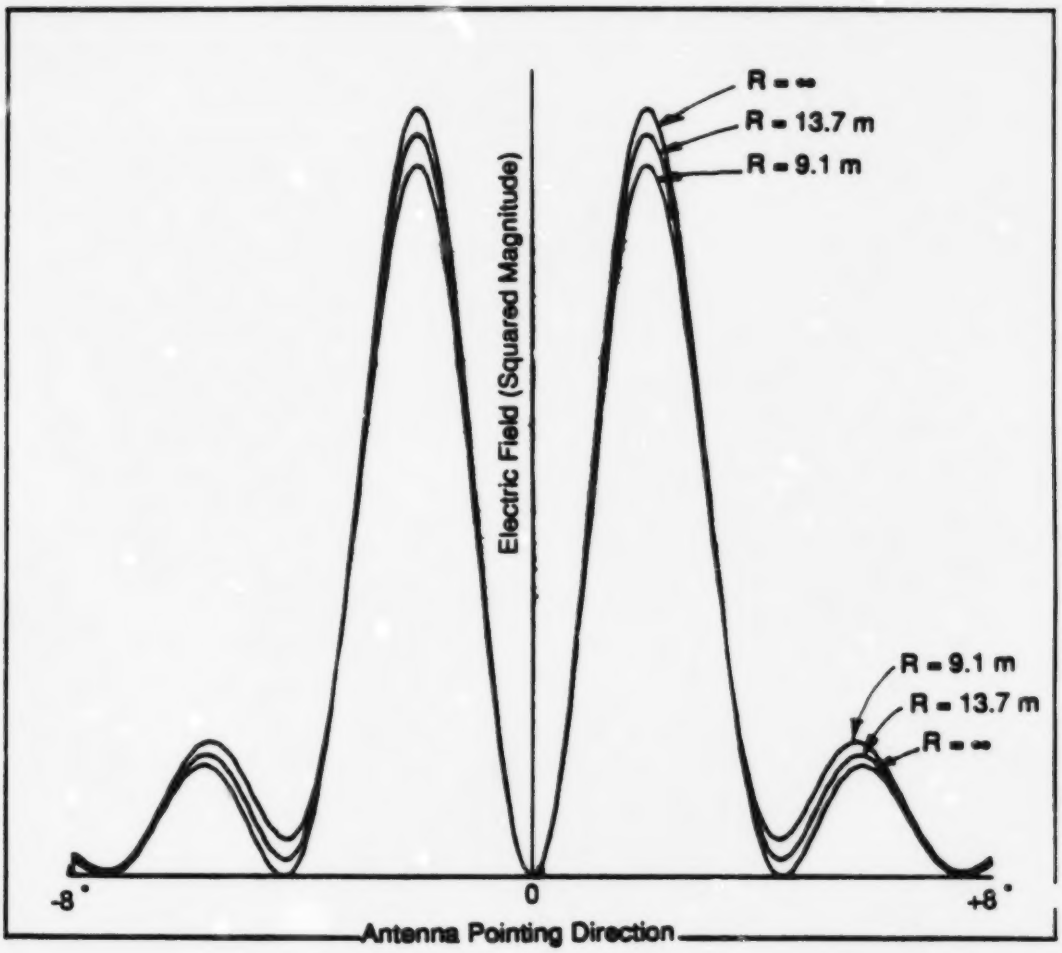

FIG. B-2: SQUARED MAGNITUDE OF THE DIFFERENCE CHANNEL RESPONSE AT VARIOUS RANGES BETWEEN RADAR AND A SINGLE POINT SOURCE

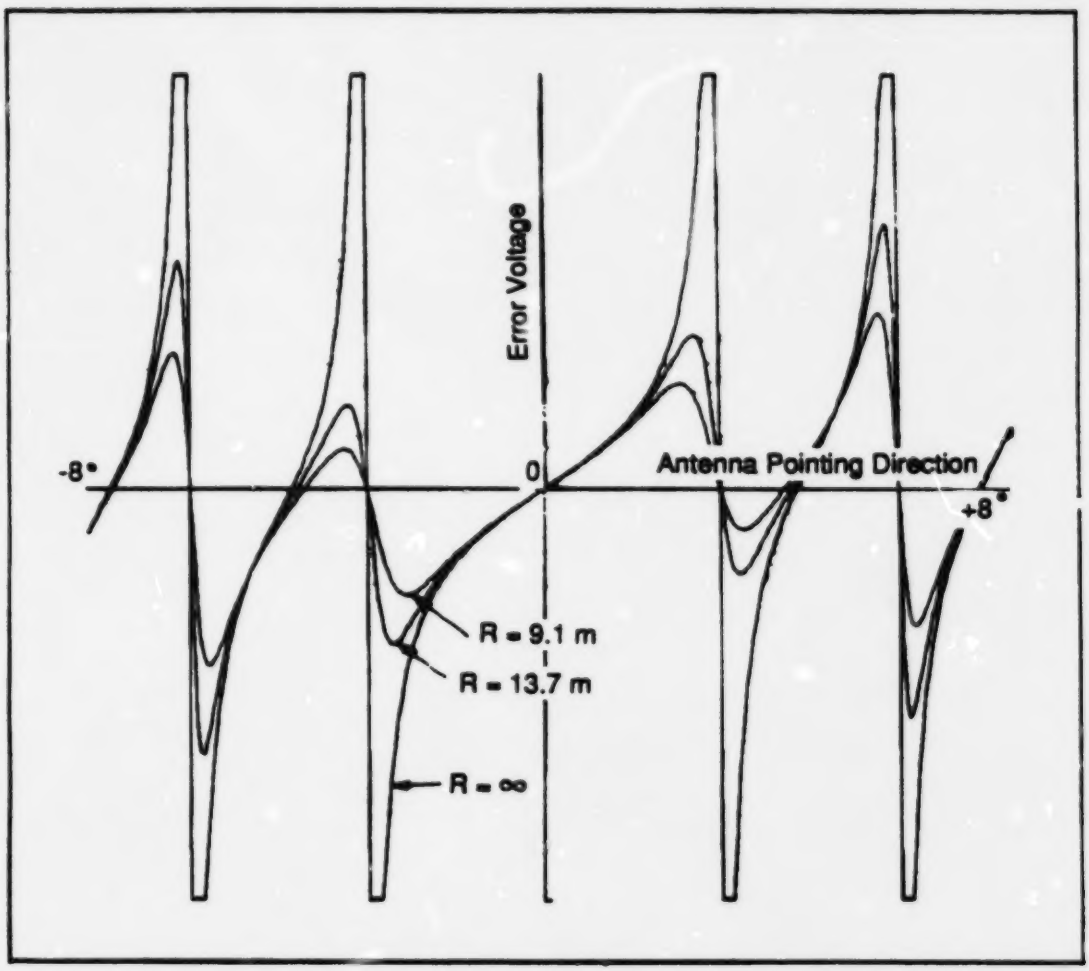

FIG B-3: MONOPULSE DISCRIMINATOR CHARACTERISTIC FOR VARIOUS RANGES BETWEEN THE RADAR AND A SINGLE POINT SOURCE

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